STUDENT NAME: ______________________________

TEACHER: ____________________________________

HURLSTONE
AGRICULTURAL
HIGH
SCHOOL

2019

HIGHER
SCHOOL
CERTIFICATE
HSC Task 4 – Trial HSC

Mathematics

Examiners

- Mr D Potaczala Ms P Biczo, Ms. T Tarannum, Ms D Crancher, Mr J Dillon

General Instructions

- Reading time – 5 minutes
- Working time – 3 hours
- Write using black pen
- NESA-approved calculators may be used
- A Reference sheet is provided
- In Questions 11 – 16, show relevant mathematical reasoning and/or calculations

Total marks: 100

Section I – 10 marks (pages 3 – 8)

- Attempt Questions 1 – 10
- Allow about 15 minutes for this section

Section II – 90 marks (pages 9 – 17)

- Attempt Questions 11 – 16
- Allow about 2 hours and 45 minutes for this section

SECTION I

10 marks

Attempt Questions 1 – 10

Allow about 10 minutes for this sections.

Use the multiple-choice answer sheet for Questions 1 – 10

Question 1

What is the value of $\frac{e^2}{6}$, correct to 3 significant figures?

A. 1.231

B. 1.232

C. 1.23

D. 1.22

Question 2

What is the solution to the equation $\cos 2x = \frac{1}{2}$ in the domain $-\pi \le x \le \pi$?

A. $x = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{-5\pi}{6}, \frac{-\pi}{6}$

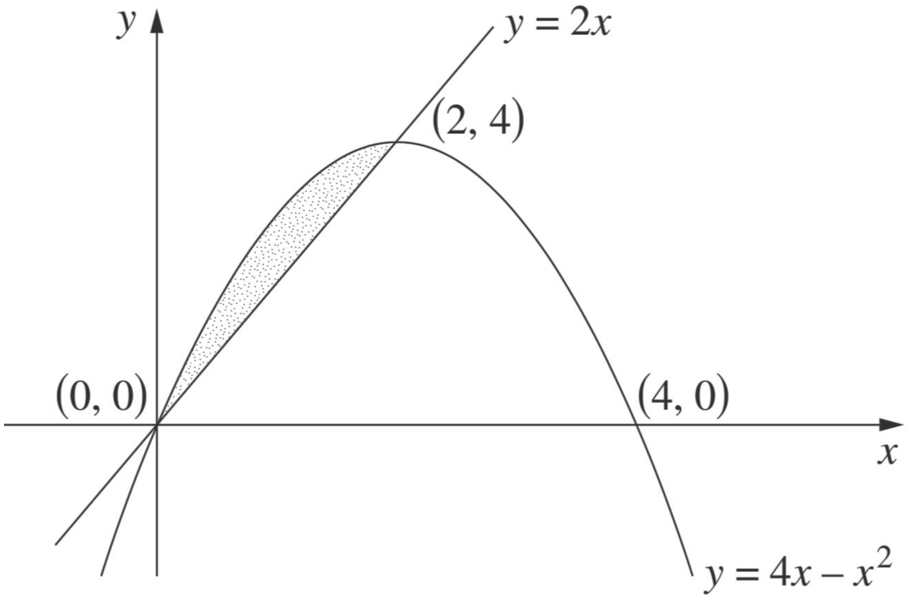
B. $x = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}$

C. $x = \frac{\pi}{12}, \frac{11\pi}{12}, \frac{13\pi}{12}, \frac{23\pi}{12}$

D. $x = \frac{\pi}{12}, \frac{11\pi}{12}, \frac{-11\pi}{12}, \frac{-\pi}{12}$

Question 3

The diagram shows the parabola $y = 4x - x^2$ meeting the $y = 2x$ line at $(0, 0)$ and $(2, 4)$.



Which expression gives the area of the shaded region bounded by the parabola and the line?

A. $\int_0^4 x^2 - 2x \, dx$

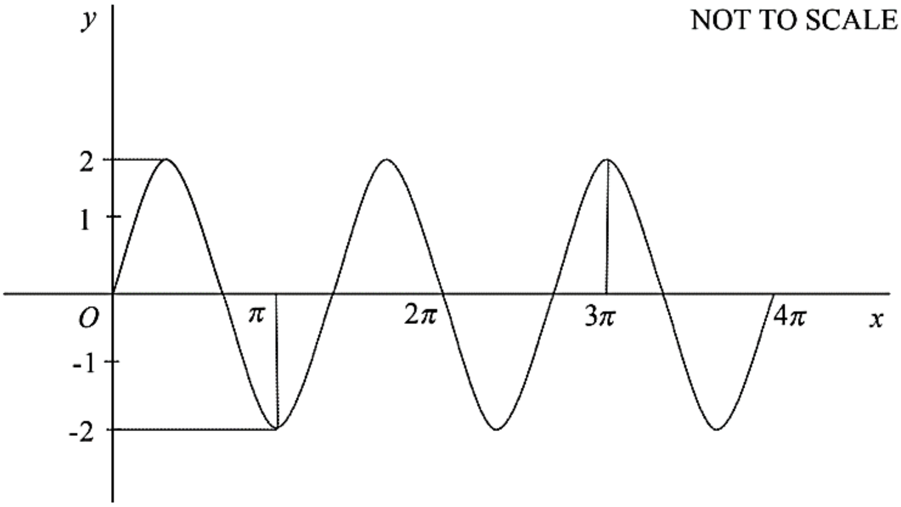
B. $\int_0^4 2x - x^2 \, dx$

C. $\int_0^2 x^2 - 2x \, dx$

D. $\int_0^2 2x - x^2 \, dx$

Question 4

Which trigonometric equation represents the graph given below?



A. $y = 2\sin\left(\frac{2\pi}{3}x\right)$

B. $y = 2\sin\left(\frac{3\pi}{2}x\right)$

C. $y = 2\sin\left(\frac{2}{3}x\right)$

D. $y = 2\sin\left(\frac{3}{2}x\right)$

Question 5

The derivative of the curve $f(x) = 4^x$ is given by which of the following?

A. $f'(x) = 4 \times 4^x$

B. $f'(x) = \log_e 4 \times 4^x$

C. $f'(x) = 4 \times e^x$

D. $f'(x) = \log_e 4 \times e^x$

Question 6

$\int e^{4x+1}\,dx$ is given by which of the following?

A. $\dfrac{e^{4x+1}}{\ln 4}+c$

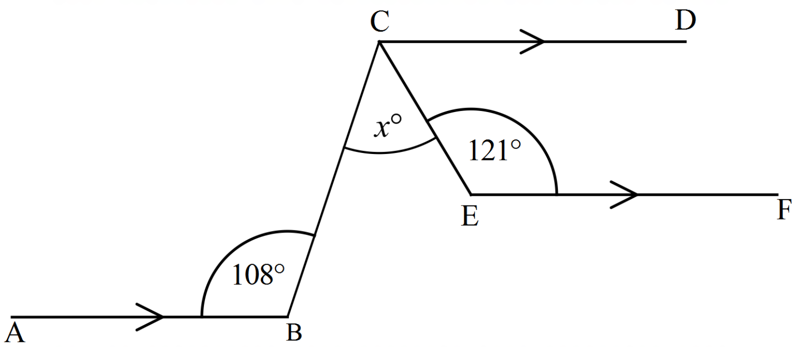
B. $\dfrac{e^{4x+1}}{4}+c$

C. $e^{4x+1}+c$

D. $4e^{4x+1}+c$

Question 7

In the diagram below, what is the value of x?



A. 49°

B. 59°

C. 108°

D. 121°

Question 8

In a class of 30 students, 8 have brown hair, 12 have brown eyes and 4 have both.

What is the probability that a student selected at random has either brown hair or brown eyes but not both?

A. $\frac{2}{15}$

B. $\frac{2}{5}$

C. $\frac{8}{15}$

D. $\frac{2}{3}$

Question 9

A flat circular disc is being heated so that the rate of increase of the area (A in m^2), after t hours, given by $\frac{dA}{dt} = \frac{1}{8}\pi t$. Initially the disc has a radius of 2 metres.

Which of the following is the correct expression for the area after t hours?

A. $A = \frac{1}{8}\pi t^2$

B. $A = \frac{1}{16}\pi t^2$

C. $A = \frac{1}{8}\pi t^2 + 4\pi$

D. $A = \frac{1}{16}\pi t^2 + 4\pi$

Question 10

It is assumed that the number $N(t)$ of ants in a certain nest at time $t \geq 0$ is given by

$$N(t) = \frac{A}{1+e^{-t}}$$

where A is a constant and t is measured in months.

At time $t = 0$, $N(t)$ is estimated at 2×10^5 ants. What is the value of A?

A. 2×10^5

B. 2×10^{-5}

C. 4×10^5

D. 4×10^{-5}

SECTION II

90 marks

Attempt Questions 11 – 16

Allow about 2hour 45 minutes for this section

Answer each question in a **SEPARATE** writing booklet. Additional writing booklets are available.

In Questions 11 – 16, your responses should include relevant mathematical reasoning and/or calculations.

Question 11 (15 marks) Start a new booklet

(a) Solve $2^{2x+1} = 32$. **2**

(b) Solve $|x-2| \leq 3$. **2**

(c) Factorise fully $36 - x^2$. **2**

(d) A parabola has a directrix $y = -2$ and a focus $(5,4)$. Find the coordinates of the vertex. **2**

(e) (i) Rationalise the denominator in the expression: **2**

$$\frac{1}{\sqrt{n}+\sqrt{n+1}}$$

where n is an integer and $n \geq 1$.

(ii) Using your result from part (i), or otherwise, find the value of the sum: **2**

$$\frac{1}{\sqrt{1}+\sqrt{2}}+\frac{1}{\sqrt{2}+\sqrt{3}}+\frac{1}{\sqrt{3}+\sqrt{4}}+\ldots+\frac{1}{\sqrt{99}+\sqrt{100}}$$

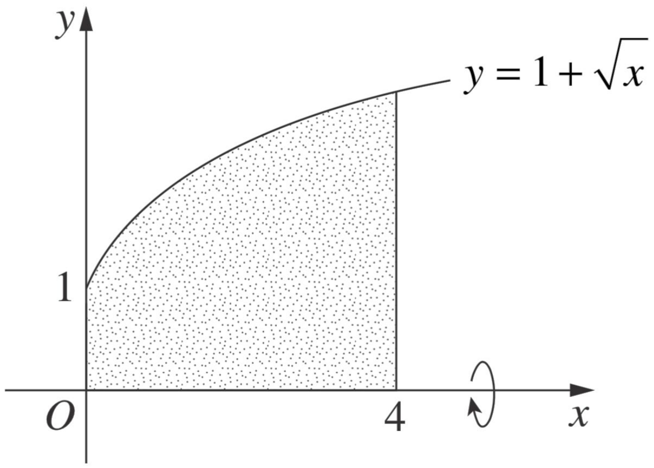
(f) A point $P(x,y)$ moves so that the sum of the squares of the distances from each of the points $A(-1,0)$ and $B(3,0)$ is equal to 40. **3**
Show that the locus of $P(x,y)$ is a circle and state its radius and centre.

Question 12 (15 marks) Start a new booklet

(a) Evaluate $\lim_{x \to 2} \frac{x^3 - 8}{x - 2}$. **2**

(b) Differentiate $y = \frac{x - 1}{3x^2}$. **2**

(c) The region bounded by the $y = 1 + \sqrt{x}$ and the x-axis between $x = 0$ and $x = 4$ is rotated about the x-axis to form a solid. Find the volume of the solid. **3**



(d) Consider the curve $y = x^3 - x^2 - x + 1$.

(i) Find the stationary points and determine their nature. **4**

(ii) Given the point $P\left(\frac{1}{3}, \frac{16}{27}\right)$ lies on the curve, prove that it is a point of inflection. **2**

(iii) Sketch the curve labelling the stationary points, point of inflection and y-intercept. **2**

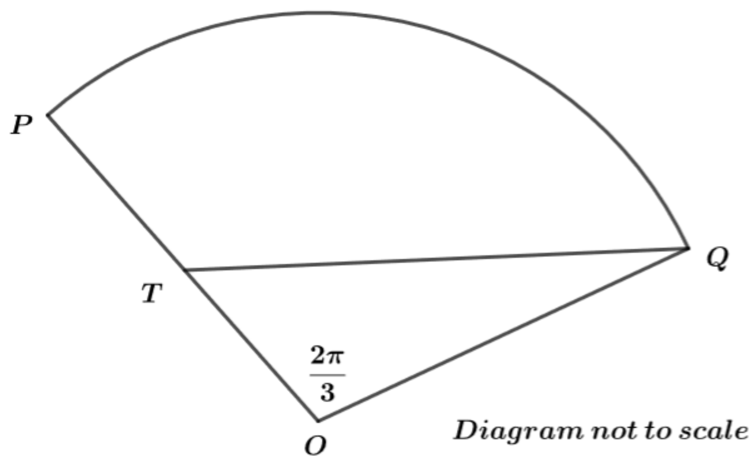
Question 13 (15 marks) Start a new booklet

(a) Solve $\tan x = \frac{1}{3}$, where $0 \le x \le 2\pi$. **2**

Give your answer(s) in radians to two decimal places.

(b) In the diagram, PQ is an arc of a circle with radius 10 cm and centre O. **3**

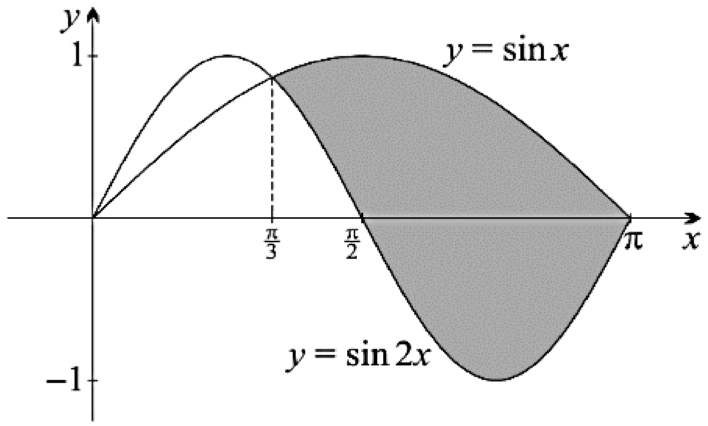
T is the midpoint of OP. Angle POQ is $\frac{2\pi}{3}$.



Find the perimeter of the shape PTQ in exact form.

Question 13 continues on next page

(c) **3**

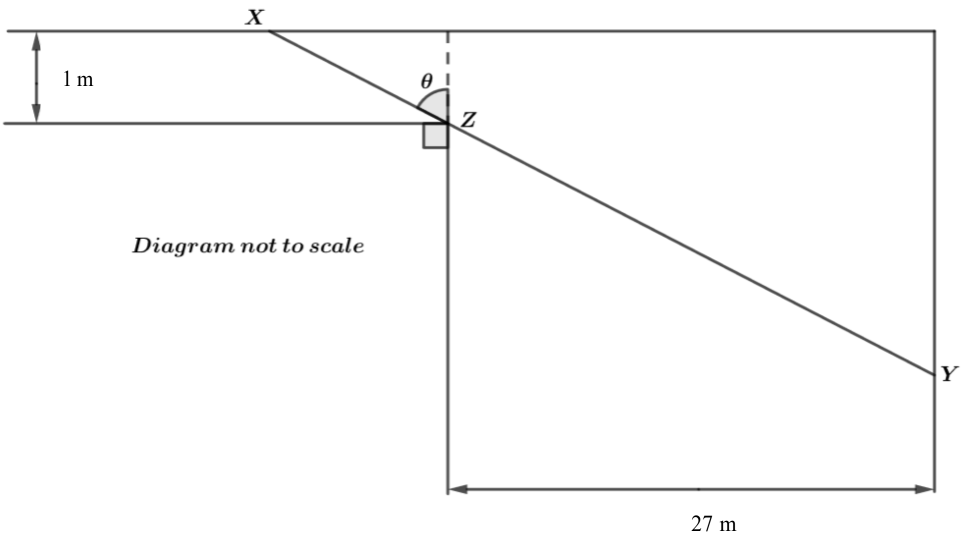


The diagram above shows the curves $y = \sin x$ and $y = \sin 2x$ for $0 \le x \le \pi$,

intersecting at $x = 0, x = \dfrac{\pi}{3}$ and $x = \pi$.

Find the exact area of the shaded region bounded by the two curves.

(d) A corridor of width 1 m enters a room of width 27 m, intersecting in a right angle, as shown in the diagram. A straight rope *XY* touches the corner *Z* as shown in the diagram below. Let *L* be the length of *XY*.



(i) Show that $L = \dfrac{1}{\cos\theta} + \dfrac{27}{\sin\theta}$. **1**

(ii) Show that $\dfrac{dL}{d\theta} = \dfrac{\cos\theta}{\sin^2\theta}\left(\tan^3\theta - 27\right)$. **3**

(iii) Given that $0 < \theta < \dfrac{\pi}{2}$, find the minimum value of *L*. **3**

Give your answer as an exact value.

Question 14 (15 marks) Start a new booklet

(a) Solve for k: $\log_5 8 = \log_5 k - 2\log_5 3$. **2**

(b) Use the trapezoidal rule, with five function values to find the area bounded by the curve $y = \ln\left(x^2 - 1\right)$, the x axis and the lines $x = 2$ and $x = 4$. Give your answer to 2 decimal places. **3**

(c) Find $\int \frac{1}{2x-1}\,dx$ **2**

(d) Given $f(x) = \log_e \sqrt{2-x}$, find using the log laws, or otherwise, $f'(x)$. **2**

(e) (i) Sketch the curve $y = \log_e(x-1)$ and shade the area bounded by the curve, the x axis, the y axis and the line $y = \log_e 5$. **1**

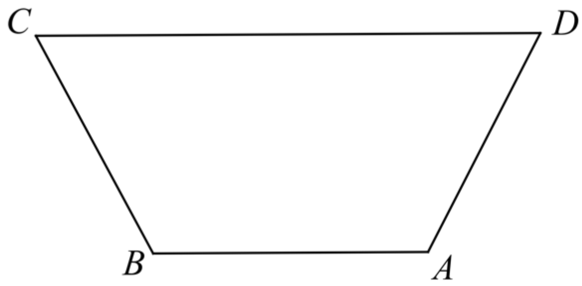
(ii) Calculate the shaded area, giving your answer in simplest exact form. **3**

(f) (i) Show that $\frac{d}{dx}\left(x e^x\right) = x e^x + e^x$. **1**

(ii) Hence, find $\int x e^x \, dx$. **1**

Question 15 (15 marks) Start a new booklet

(a) *ABCD* is a quadrilateral with $\angle ABC = \angle BAD$ and $BC = AD$.



Not to scale

(i) Prove that $\Delta ABC \equiv \Delta BAD$. **2**

(ii) Why are $\angle CAB$ and $\angle ABD$ equal? **1**

(iii) Prove that $\angle DBC = \angle CAD$. **2**

(b) A triangle has vertices $A(1, -3)$, $B(3, 3)$ and $C(-3, 1)$.

(i) Find the coordinates of *L* and *M*, the midpoints of *AB* and *BC* respectively. **1**

(ii) Show that *LM* is parallel to *AC* and that $LM = \frac{1}{2}AC$. **2**

(c) The senior students at a school decide to send a delegation of two to a conference.

The delegation will have one student from Year 11 and one from Year 12.

The candidates from Year 11 are Petra, Quentin and Rufus, who have probabilities of $\frac{1}{6}$, $\frac{1}{3}$ and $\frac{1}{2}$ respectively of being chosen by their classmates.

The candidates from Year 12 are Amelia, Bella, Charles and Diana, who have probabilities of $\frac{1}{8}$, $\frac{1}{4}$, $\frac{3}{8}$ and $\frac{1}{4}$ respectively of being chosen.

(i) Draw a tree diagram to show the possible pairs in the delegation. Include the probability on each branch. **2**

(ii) What is the probability that the delegation includes either Quentin or Bella but not both? **1**

Question 15 continues on next page

(d) A worker invests $\$P$ at the beginning of each month into a retirement fund that pays 6% p.a. compounded monthly, on the money invested, for 20 years.

(i) Show that after 2 months there is $\$P(1.005^2 + 1.005)$ in the fund. **1**

(ii) Write an expression for the amount in the fund after one year. **1**

(iii) The worker wishes to retire at the end of the 20 years with a lump sum of \$450 000. **2**

What investment must the worker make at the beginning of each month to achieve this?

Question 16 (15 marks) Start a new booklet

(a) A water tank has an initial capacity of 3000 litres and is leaking according to the formula $V = V_0 e^{-kt}$, where t is in hours.

(i) Show that $\dfrac{dV}{dt} = -kV$. **1**

(ii) What is the value of k if after 3 hours the volume is 2000 litres? Give your answer correct to 3 decimal places. **2**

(iii) How long will it take for the amount of water in the tank to fall to 250 litres? Give your answer correct to the nearest minute. **2**

(b) The rate of emission of carbon pollution C, in tonnes per year from a factory from 1st January 2011 is given by:

$$C = 500 - \left(\frac{10}{1+t}\right)^2 \text{ where } t \text{ is the time in years.}$$

(i) What was the rate of emission of carbon pollution C on 1st January 2011? **1**

(ii) What was the rate of emission of carbon pollution C on 1st January 2013? **1**

(iii) What value does C approach as time passes? **1**

(iv) Draw a sketch of C as a function of t. **1**

(v) Calculate the total amount of carbon pollution emitted from the factory from 1st January 2011 to 1st January 2017? Answer correct to the nearest tonne. **2**

Question 16 continues on next page

(c) A factory produces mobile phones. The annual production of phones, M, at time t years, is given by:

$$M = 2000e^{kt} \text{ where } k \text{ is a constant.}$$

After five years, the production has increased to 3200 phones per annum.

(i) Find the value of k. **1**

(ii) What is the predicted production after 10 years? **1**

(iii) How many years will it take for the production to double its original output? **1**

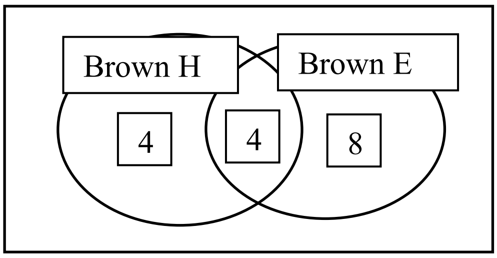
(iv) Find the rate of increase in production when the factory has been operating for 5 years. **1**

End of Paper

Outcomes Addressed in this Question

PE2 Uses multi-step deductive reasoning in a variety of contexts
PE6 Makes comprehensive use of mathematical language, diagrams & notation for communicating in a wide variety of situations
HE1 Appreciates interrelationships between ideas drawn from different areas of mathematics
HE7 evaluates mathematical solutions to problems and communicates them in an appropriate form

Outcome	**Solutions**	**Marking Guidelines**
PE2	Q1. $\frac{e^2}{6} = 1.2315\ldots$ 3 significant figures is **1.23**.	1 mark
PE2	Q2. $\cos 2x = \frac{1}{2}$ in the domain $-\pi \leq x \leq \pi$. Since it is $2x$, change the domain to $-2\pi \leq 2x \leq 2\pi$. $2x = \cos^{-1}\left(\frac{1}{2}\right)$ $2x = -\frac{5\pi}{3}, -\frac{\pi}{3}, \frac{\pi}{3}, \frac{5\pi}{3}$ $x = -\frac{5\pi}{6}, -\frac{\pi}{6}, \frac{\pi}{6}, \frac{5\pi}{6}$	1 mark
PE2	Q3. *The parabola* $y = 4x - x^2$ *is above the line* $y = 2x$. $\int_0^2 4x - x^2 - 2x \ \ dx$ $= \int_0^2 \mathbf{2x - x^2 \ \ dx}$	1 mark
PE2	Q4. The graph has, amplitude=2 and period = $\frac{3}{2}$ $\therefore y = 2\sin(\frac{3}{2}x)$	1 mark
PE6	Q5. $\frac{d}{dx}(a^x) = a^x \ln a$ $f(x) = 4^x$ $f'(x) = 4^x \ln 4$ $\boldsymbol{f'(x) = 4^x \times \log_e 4}$	1 mark

PE6	Q6. $\int\left(e^{f(x)}\right) = \frac{1}{f'(x)}e^{f(x)} + c$ $\int e^{4x+1}\, dx$ $f(x) = 4x + 1 \text{ and } f'(x) = 4$ $\therefore \frac{e^{4x+1}}{4} + c$	1 mark
PE6	Q7. $\angle ABC = 108° \ (given)$ $\angle BCD = 108° \ (alternative\ angles\ on\ parallel\ lines)$ $\angle CEF = 121° \ (given)$ $\angle ECD = 180° - 121° =$ $59° \ (cointerior\ angles\ on\ parallel\ lines)$ $\angle BCD - \angle ECD = \angle BCE$ $\therefore 108° - 59° = \mathbf{49}°$	1 mark
PE2	Q8  There are 4 students who only have brown hair and 8 students who only have brown eyes out of the 30 students. $\therefore \frac{12}{30} = \mathbf{\frac{2}{5}}$	1 mark
PE2	Q9. $\frac{dA}{dt} = \frac{1}{8}\pi t$ $A = \int \frac{1}{8}\pi t\, dt$ $= \frac{1}{8} \times \frac{1}{2} \times \pi \times t^2 + c$ To find the area of a circle, we can replace $A = \pi r^2$ $\pi r^2 = \frac{1}{16}\pi t^2 + c$ At t=0, r=2 $4\pi = 0 + c$ $c = 4\pi$ $\therefore A = \frac{1}{16}\pi t^2 + 4\pi$	1 mark

HE7, HE1	Q10. At time $t = 0$, $N(t) = \frac{A}{1+e^{-0}} = \frac{A}{1+1} = \frac{A}{2}$ $N(t)$ is estimated at 2×10^5. $2 \times 10^5 = \frac{A}{2}$ $A = 2 \times 2 \times 10^5$ $\boldsymbol{A = 4 \times 10^5}$	1 mark

Year 12 Mathematics Trial 2019		
Question No. 11	Solutions and Marking Guidelines	
Outcomes Addressed in this Question		
P3 - performs routine arithmetic and algebraic manipulation involving surds, simple rational expressions and trigonometric identities P4 - chooses and applies appropriate arithmetic, algebraic, graphical and geometric techniques		
Outcome	**Solutions**	**Marking Guidelines**
P3	(a) $2^{2x+1} = 32$ $2^{2x+1} = 2^5$ $2x+1 = 5$ $x = 2$	**2 marks** correct solution **1 mark** error made
P3	(b) $\lvert x-2 \rvert \le 3$ $-3 \le x-2 \le 3$ $-1 \le x \le 5$	**2 marks** correct solution **1 mark** error made
P3 P4	(c) $36 - x^2 = 6^2 - x^2$ $= (6-x)(x+x)$	**2 marks** correct solution **1 mark** error made
P3 P4	(d) y-coordinate $= \frac{-2+4}{2} = 1$ focus $(5,1)$	**2 marks** correct solution **1 mark** error made
		2 marks correct solution **1 mark** error made

P3 P4	(e) (i) $$\frac{1}{\sqrt{n}+\sqrt{n+1}}=\frac{1}{\sqrt{n}+\sqrt{n+1}}\times\frac{\sqrt{n}-\sqrt{n+1}}{\sqrt{n}-\sqrt{n+1}}$$ $$=\frac{\sqrt{n}-\sqrt{n+1}}{n-(n+1)}$$ $$=\frac{\sqrt{n}-\sqrt{n+1}}{-1}$$ $$=-\sqrt{n}+\sqrt{n+1}$$	**2 marks** correct solution **1 mark** error made
P3 P4	(ii) $$\frac{1}{\sqrt{1}+\sqrt{2}}+\frac{1}{\sqrt{2}+\sqrt{3}}+...+\frac{1}{\sqrt{99}+\sqrt{100}}=\sqrt{2}-\sqrt{1}+\sqrt{3}-\sqrt{2}+...+\sqrt{100}-\sqrt{99}$$ $$=\sqrt{100}-\sqrt{1}$$ $$=9$$	**3 marks** correct solution **2 marks** error made **1 mark** establishes correct relationship of distances
P3 P4	(f) $$PA^2+PB^2=40$$ $$(x-(1))^2+(y-0)^2+(x-3)^2+(y-0)^2=40$$ $$x^2+2x+1+y^2+x^2-6x+9+y^2=40$$ $$2x^2+2y^2-4x=30$$ $$x^2+y^2-2x=15$$ $$x^2-2x+1+y^2=15+1$$ $$(x-1)^2+y^2=16$$ Radius $=4$ Centre $(1,0)$	

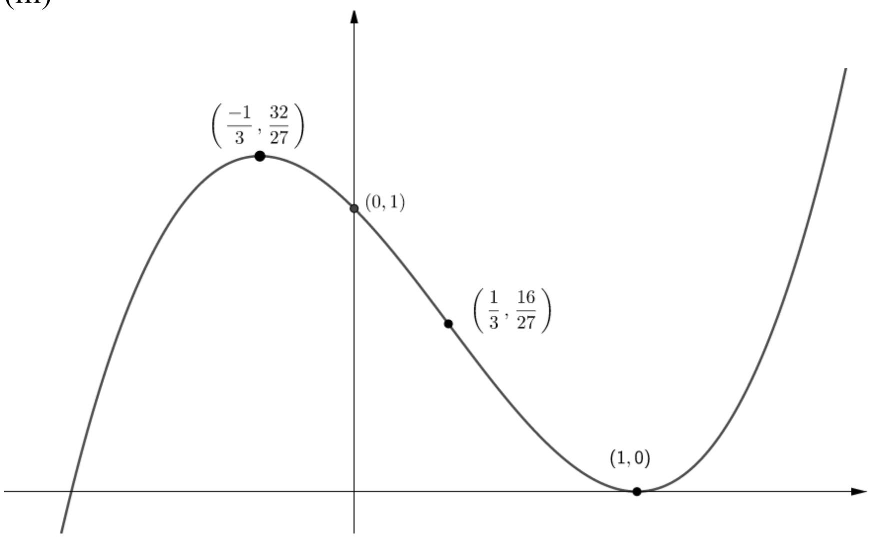
Year 12 Mathematics Trial 2019		
Question No. 13	Solutions and Marking Guidelines	
Outcomes Addressed in this Question		
P4 - chooses and applies appropriate arithmetic, algebraic, graphical and geometric techniques P5 - understands the concept of a function and the relationship between a function and its graph H6 - uses the derivative to determine the features of the graph of a function P6 - relates the derivative of a function to the slope of its graph P7 - determines the derivative of a function through routine application of the rules of differentiation H7 - uses the features of a graph to deduce information about the derivative H8 - uses techniques of integration to calculate areas and volumes		
Outcome	**Solutions**	**Marking Guidelines**
P4	(a) $$\lim_{x\to 2}\frac{x^3-8}{x-2}=\lim_{x\to 2}\frac{(x-2)(x^2+2x+4)}{x-2}$$ $$=\lim_{x\to 2}x^2+2x+4$$ $$=12$$	**2 marks** correct solution **1 mark** error made

P7	(b) $$y=\frac{x-1}{2x^2}$$ $$y'=\frac{3x^2(1)-6x(x-1)}{(3x^2)^2}$$ $$=\frac{6x-3x^2}{9x^2}$$ $$=\frac{2-x}{3x^3}$$	**2 marks** correct solution **1 mark** error made
H8	(c) $$V=\pi\int_0^4 y^2dx$$ $$=\pi\int_0^4\left(1+\sqrt{x}\right)^2dx$$ $$=\pi\int_0^4 1+2x^{\frac{1}{2}}+x\ dx$$ $$=\pi\left[x+\frac{2x^{\frac{3}{2}}}{3/2}+\frac{x^2}{2}\right]_0^4$$ $$=\pi\left(4+\frac{4}{3}\times4\times2+8-0\right)$$ $$=\frac{68\pi}{3}\text{ units}^3$$	**3 marks** correct solution **2 marks** error made **1 mark** demonstrates some relevant knowledge about volumes and integration
H6	(d) (i) $$y=x^3-x^2-x+1$$ $$y'=3x^2-2x-1$$ $$0=3x^2-2x-1$$ $$0=(3x+1)(x-1)$$ $$x=\frac{-1}{3},1$$ At $\left(-\frac{1}{3},\frac{32}{27}\right)$ $\quad y''=6\left(-\frac{1}{3}\right)-2=-4<0$ Therefore maximum At $(1,0)$ $\quad y''=6(1)-2=4>0$ Therefore minimum	**4 marks** correct solution **3 marks** error made **2 marks** finds correct *x*-values of stationary points **1 mark** demonstrates some relevant knowledge about using calculus to find stationary points

<table>
<tr>
<td>P6
H7</td>
<td>
(ii)

<table>
<tr><td>x</td><td>0</td><td>$\frac{1}{3}$</td><td>$\frac{1}{2}$</td></tr>
<tr><td>y''</td><td>-2</td><td>0</td><td>1</td></tr>
</table>

There is a change in concavity at $x = \frac{1}{3}$, since there is a change in sign. Therefore $\left(\frac{1}{3}, \frac{16}{27}\right)$ is a point of inflection
</td>
<td>2 marks
correct solution with concavity checked
1 mark
error made</td>
</tr>
<tr>
<td>H6
H9</td>
<td>
(iii)



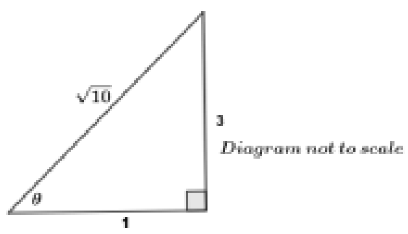
</td>
<td>2 marks
correct graph, including curvature and y-intercept
1 mark
error made</td>
</tr>
</table>

Year 12 Question 13	Mathematics Solutions and Marking Guidelines	Trial HSC (Task 4) 2019

Outcome Addressed in this Question

H5 **applies appropriate techniques from the study of** calculus, geometry, probability, **trigonometry** and series **to solve problems**

Part	Solutions	Marking Guidelines
(a)	$\tan x = \frac{1}{3}$ $x = 0.32$ or 3.46 (to 2 d.p)	**Award 2** for correct solutions **Award 1** for substantial progress towards solution **or** answers in degrees i.e. $x = 18.43^0, 198.43^0$ or $18^0 26', 198^0 26'$
(b)	Length of arc $PQ = 10 \times \frac{2\pi}{3} = \frac{20\pi}{3}$ Length $TQ = \sqrt{5^2 + 10^2 - 2 \times 5 \times 10 \times \cos\left(\frac{2\pi}{3}\right)}$ $= \sqrt{25 + 100 - 100 \times \left(-\frac{1}{2}\right)}$ $= \sqrt{125 + 50}$ $= \sqrt{175} = \sqrt{25 \times 7} = 5\sqrt{7}$ Perimeter PTQ = length of arc PQ + length PT + length TQ $= \left(\frac{20\pi}{3} + 5 + 5\sqrt{7}\right)$ cm	**Award 3** for correct solution **Award 2** for substantial progress towards solution **Award 1** for limited progress towards solution
(c)	$A = \int_{\frac{\pi}{3}}^{\pi} (\sin x - \sin 2x)\,dx$ $= \left[-\cos x + \frac{1}{2}\cos 2x\right]_{\frac{\pi}{3}}^{\pi}$ $= \left[-\cos \pi + \frac{1}{2}\cos 2\pi\right] - \left[-\cos\frac{\pi}{3} + \frac{1}{2}\cos 2\left(\frac{\pi}{3}\right)\right]$ $= \left[1 + \frac{1}{2}\right] - \left[-\frac{1}{2} + \frac{1}{2}\left(-\frac{1}{2}\right)\right]$ $= 1 + \frac{1}{2} + \frac{1}{2} + \frac{1}{4} = 2\frac{1}{4}$ square units	**Award 3** for correct solution **Award 2** for substantial progress towards solution **Award 1** for limited progress towards solution
(d) (i)	L = length $XY = XZ + YZ$ $XZ = \frac{1}{\cos\theta}$ $YZ = \frac{27}{\sin\theta}$ $\therefore L = \frac{1}{\cos\theta} + \frac{27}{\sin\theta}$	**Award 1** for correct solution

(ii)	$L=\dfrac{1}{\cos\theta}+\dfrac{27}{\sin\theta}$ $\dfrac{dL}{d\theta}=\dfrac{-1(-\sin\theta)}{\cos^2\theta}+\dfrac{-27\cos\theta}{\sin^2\theta}$ $=\dfrac{\sin\theta}{\cos^2\theta}-\dfrac{27\cos\theta}{\sin^2\theta}$ $=\dfrac{\sin^3\theta-27\cos^3\theta}{\cos^2\theta\sin^2\theta}$ $=\dfrac{1}{\sin^2\theta}\left(\dfrac{\sin^3\theta-27\cos^3\theta}{\cos^2\theta}\right)$ $=\dfrac{1}{\sin^2\theta}\times\dfrac{\cos\theta}{\cos\theta}\left(\dfrac{\sin^3\theta-27\cos^3\theta}{\cos^2\theta}\right)$ $=\dfrac{\cos\theta}{\sin^2\theta}\left(\dfrac{\sin^3\theta-27\cos^3\theta}{\cos^3\theta}\right)$ $=\dfrac{\cos\theta}{\sin^2\theta}\left(\dfrac{\sin^3\theta}{\cos^3\theta}-\dfrac{27\cos^3\theta}{\cos^3\theta}\right)$ $\therefore\dfrac{dL}{d\theta}=\dfrac{\cos\theta}{\sin^2\theta}\left(\tan^3\theta-27\right)$	**Award 3** for correct solution **Award 2** for substantial progress towards solution **Award 1** for limited progress towards solution
(iii)	$\dfrac{dL}{d\theta}=\dfrac{\cos\theta}{\sin^2\theta}\left(\tan^3\theta-27\right)$ for minimum value $\dfrac{dL}{d\theta}=0$ $\dfrac{\cos\theta}{\sin^2\theta}\left(\tan^3\theta-27\right)=0$ $\dfrac{\cos\theta}{\sin^2\theta}=0$ or $\left(\tan^3\theta-27\right)=0$ $\cos\theta=0$ No solution as $0<\theta<\dfrac{\pi}{2}$ $\tan^3\theta=27$ $\tan\theta=3$  	**Award 3** for correct solution **Award 2** for substantial progress towards solution **Award 1** for limited progress towards solution

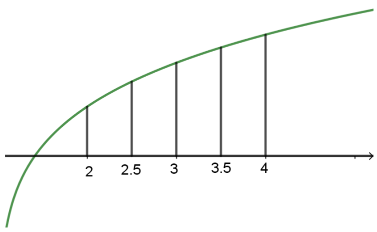
	$\sin\theta = \dfrac{3}{\sqrt{10}}$ and $\cos\theta = \dfrac{1}{\sqrt{10}}$ $L = \dfrac{1}{\frac{1}{\sqrt{10}}} + \dfrac{27}{\frac{3}{\sqrt{10}}}$ $L = \sqrt{10}\left(\dfrac{1}{1} + \dfrac{27}{3}\right)$ $L = \sqrt{10}\left(\dfrac{30}{3}\right)$ $\therefore L = 10\sqrt{10}$ units	

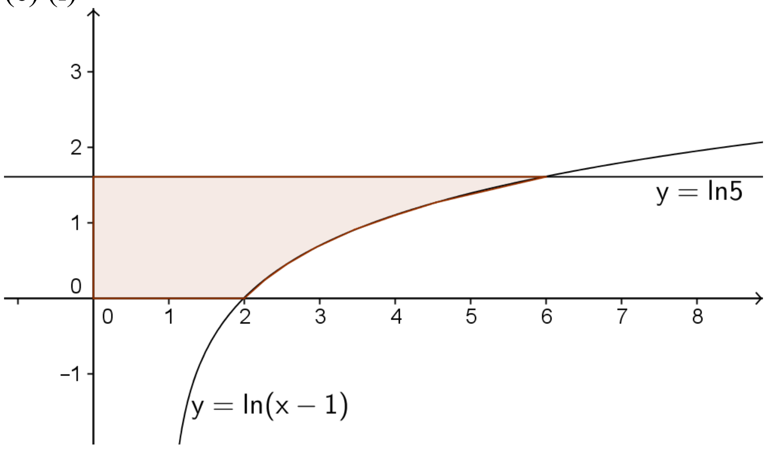
Year 12 Mathematics Trial 2019

Question No. 14 Solutions and Marking Guidelines

Outcomes Addressed in this Question

H3 Manipulates algebraic expressions involving logarithmic & exponential functions
H5 Applies appropriate techniques from the study of calculus, geometry, probability, trigonometry & series to solve problems
H8 Uses techniques of integration to calculate areas & volumes
P5 Understands the concept of a function and the relationship between a function and its graph
P7 Determines the derivative of a function through routine application of the rules of differentiation

Outcome	Solutions	Marking Guidelines
H3	(a) $\log_5 8 = \log_5 k - 2\log_5 3$ $\therefore \log_5 8 = \log_5 k - \log_5 3^2$ $\therefore \log_5 8 = \log_5 \dfrac{k}{9}$ $\therefore 8 = \dfrac{k}{9}$ $k = 72$	**2 marks**: correct solution **1 mark**: substantial progress towards correct solution
H5	(b)  $f(x) = \ln(x^2 - 1)$ $A = \dfrac{0.5}{2}\{f(2) + 2[f(2.5) + f(3) + f(3.5)] + f(4)\}$ $= \dfrac{0.5}{2}\{\ln 3 + 2[\ln 5.25 + \ln 8 + \ln 11.25] + \ln 15\}$ $= 4.03$ (to 2 decimal places)	**3 marks**: correct solution **2 marks**: substantially correct solution **1 mark**: substantial progress towards correct solution
H8	(c) $\displaystyle\int \frac{1}{2x-1}\,dx = \frac{1}{2}\int \frac{2}{2x-1}\,dx$ $= \dfrac{1}{2}\ln\lvert 2x-1\rvert + c$	**2 marks**: correct solution **1 mark**: substantial progress towards correct solution

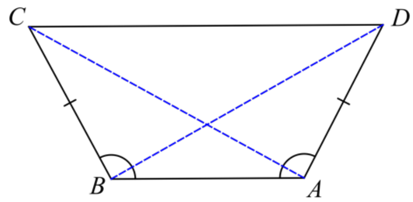
H3, H5	(d) $f(x)=\log_e\sqrt{2-x}$ $=\log_e(2-x)^{\frac{1}{2}}$ $=\frac{1}{2}\log_e(2-x)$ $f'(x)=\frac{1}{2}.\frac{-1}{2-x}$ $=\frac{-1}{2(2-x)}$	**2 marks**: correct solution **1 mark**: substantial progress towards correct solution
P5, H5	(e) (i)  	**1 mark**: correct solution
H3, H8	(ii) Shaded area $=\int_0^{\ln 5} x\,dy$ If $y=\log_e(x-1)$, $e^y=x-1$ $\therefore x=e^y+1$ $\therefore$ shaded area $=\int_0^{\ln 5}\left(e^y+1\right)dy$ $=\left[e^y+y\right]_0^{\ln 5}$ $=e^{\ln 5}+\ln 5-(e^0+0)$ $=5+\ln 5-1$ $=4+\ln 5\ \text{u}^2$	**3 marks**: correct solution **2 marks**: substantially correct solution **1 mark**: substantial progress towards correct solution
P7, H5	(f) (i) $\frac{d}{dx}\left(xe^x\right)=x\frac{d}{dx}\left(e^x\right)+e^x\frac{d}{dx}(x)$ (Product rule) $=xe^x+e^x.1$ $\therefore\frac{d}{dx}\left(xe^x\right)=xe^x+e^x$ Note: as question said 'show that' evidence was needed of How the answer was reached. This needed to be specific to this question, such as what your *u* and *v* were when using the product rule	**1 mark**: correct solution **1 mark**: correct solution

H5	(ii) Since $\frac{d}{dx}\left(xe^x\right) = xe^x + e^x$ $\int\left(xe^x + e^x\right)dx = xe^x + c$ $\therefore \int xe^x dx + \int e^x dx = xe^x + c$ $\therefore \int xe^x dx + e^x = xe^x + c$ $\therefore \int xe^x dx = xe^x - e^x + c$	

Year 12 Question 15	Mathematics Solutions and Marking Guidelines	Trial HSC (Task 4) 2019

Outcome Addressed in this Question

P2 - provides reasoning to support conclusions which are appropriate to the context
H2 - constructs arguments to prove and justify results
H5 - applies appropriate techniques from the study of calculus, geometry, probability, trigonometry and series to solve problems
H9 - communicates using mathematical language, notation, diagrams and graphs

Part	Solutions	Marking Guidelines
(a) (i) P2, H2, H5	 Consider ΔABC and ΔBAD $BC = AD$ (given) $\angle ABC = \angle BAD$ (given) $AB = AB$ (common side) $\Delta ABC \equiv \Delta BAD$ (SAS)	**Award 2** for correct solution **Award 1** for substantial progress towards solution
(ii) P2, H2	$\angle CAB = \angle ABD$ (matching angles in congruent triangles)	**Award 1** for correct solution
(iii) H2, H5	$\angle ABC = \angle BAD$ $\angle DBC + \angle ABD = \angle CAD + \angle CAB$ (adjacent angles) Since $\angle CAB = \angle ABD$ (from part (ii)) Therefore $\angle DBC = \angle CAD$	**Award 2** for correct solution **Award 1** for substantial progress towards solution
(b) (i) H5	$x = \frac{1+3}{2}, y = \frac{-3+3}{2}$ $\quad$ $x = \frac{3-3}{2}, y = \frac{3+1}{2}$ L = (2, 0) $\quad$ $M = (0, 2)$	**Award 1** for correct solutions

(ii)

H2, H5

$$m_{LM} = \frac{2-0}{0-2}$$
$$= -1$$ b

$$m_{\overrightarrow{AC}} = \frac{1+3}{-3-1}$$
$$= -1 \quad \therefore \quad LM \parallel AC$$

$$LM = \sqrt{(0-2)^2 + (2-0)^2}$$
$$= \sqrt{4+4}$$
$$= \sqrt{8}$$
$$= 2\sqrt{2}$$

$$\overrightarrow{AC} = \sqrt{(-3-1)^2 + (1+3)^2}$$
$$= \sqrt{16+16}$$
$$= \sqrt{32}$$
$$= 4\sqrt{2}$$

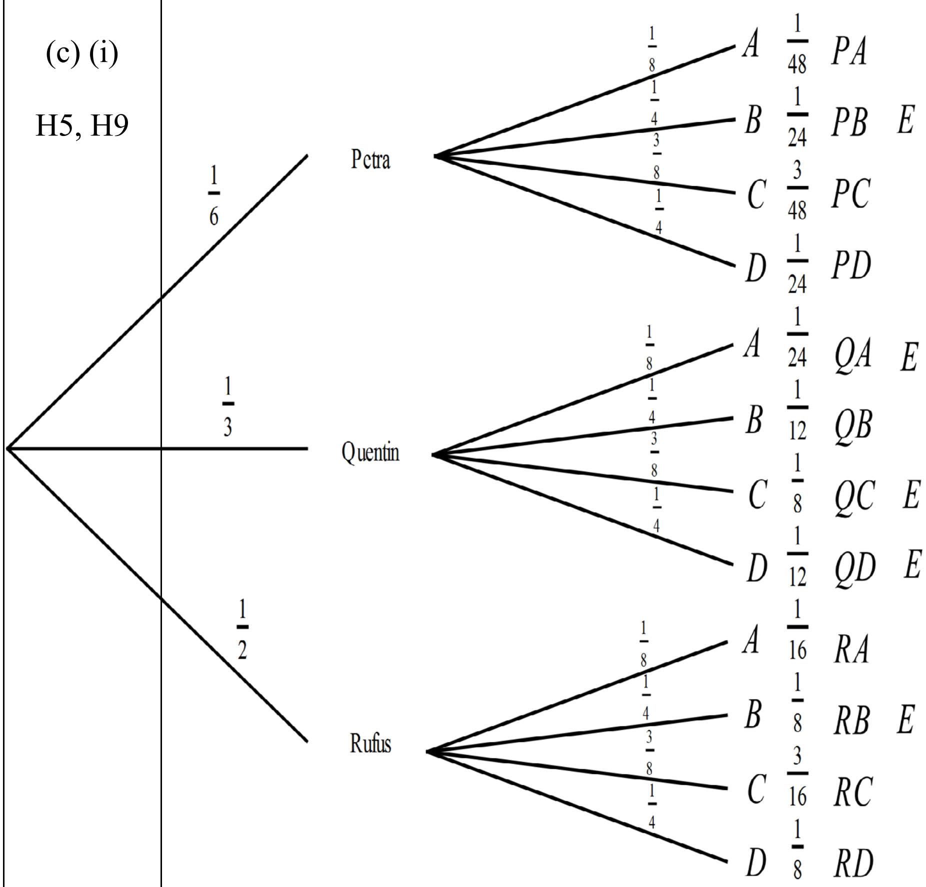
$$\therefore \; 2 \times LM = AC$$
$$LM = \frac{1}{2}AC$$

Award 2 for correct solution

Award 1 for substantial progress towards solution

(c) (i)

H5, H9



Award 2 for correct solution

Award 1 for substantial progress towards solution

(ii) H5	Outcomes are those marked E on the diagram $P(Q \text{ or } B \text{ but not both}) = \frac{1}{24}+\frac{1}{24}+\frac{1}{8}+\frac{1}{12}+\frac{1}{8}$ $= \frac{10}{24} = \frac{5}{12}$	**Award 1** for correct solution
(d) (i) H5 H1	$r = 6\%\ pa = 0.5\%$ per month $= 0.005$ $M_1 = P(1.005)$ $M_2 = (P(1.005)+P) \times 1.005$ $= P(1.005)^2 + P(1.005)$ $= P(1.005^2 + 1.005)$	**Award 1** for correct solution
(ii) H5	$M_{12} = P(1.005^{12} + 1.005^{11} + \text{...........}1.005)$	**Award 1** for correct solution
(iii) H5	$n = 12 \times 20 = 240 \quad M_{240} = \$450000$ $S_{240} = \frac{1.005(1.005^{240}-1)}{1.005-1}$ $= 464.351$ $P = \frac{450000}{464.351}$ $= \$969.09$	**Award 2** for correct solution **Award 1** for substantial progress towards solution

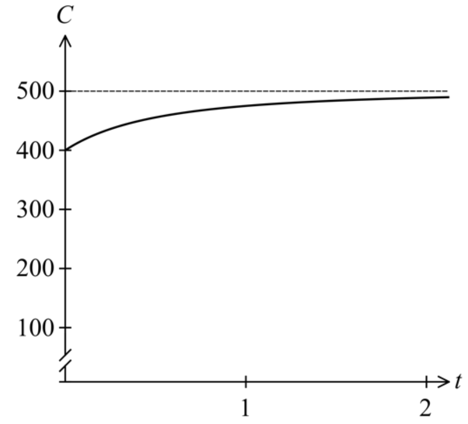
Year 12 Question16	Mathematics Solutions and Marking Guidelines	Trial HSC (Task 4) 2019

Outcome Addressed in this Question

H4 - Expresses practical problems in mathematical terms based on simple given models.
H1 - seeks to apply mathematical technique to problems in a wide range of practical context

Part	Solutions	Marking Guidelines
(a) (i)	$V = 3000e^{-kt}$ $\frac{dV}{dt} = -k \times 3000e^{-kt} = -kV$	**Award 1** for correct solution
(ii)	When $t = 3$ then $V = 2000$	**Award 2** for correct solution **Award 1** for substantial progress towards solution

(iii)	$2000 = 3000e^{-3k}$ $e^{-3k} = \frac{2}{3}$ $\log_e e^{-3k} = \log_e 0.\dot{6}$ $-3k = \log_e 0.\dot{6}$ $k = \frac{\log_e 0.\dot{6}}{-3} = 0.135155036... \approx 0.135$ $2000 = 3000e^{-3k}$ $e^{-3k} = \frac{2}{3}$ $\log_e e^{-3k} = \log_e 0.\dot{6}$ $-3k = \log_e 0.\dot{6}$ $k = \frac{\log_e 0.\dot{6}}{-3} = 0.135155036... \approx 0.135$	**Award 2** for correct solution **Award 1** for substantial progress towards solution
(b) (i)	We need to find t when $V = 250$. $250 = 3000e^{-k \times t}$ $e^{-kt} = 0.08\dot{3}$ $-kt = \log_e 0.08\dot{3}$ $t = -\frac{1}{k}\log_e 0.08\dot{3} = -\frac{\log_e 0.08\dot{3}}{0.135155..}$ $= 18.385601.. \approx 18$ h 23 min	**Award 1** for correct solution
(ii)	Initial calculation occurs on 1st January 2011 or $t = 0$ $C = 500 - \left(\frac{10}{1+0}\right)^2$ $= 400$ tonnes per year	**Award 1** for correct solution
(iii)	1st January 2013 requires $t = 2$ $C = 500 - \left(\frac{10}{1+2}\right)^2$ $= 488.\dot{8}$ tonnes per year	**Award 1** for correct solution
(iv)	$C = \lim_{t\to\infty} 500 - \left(\frac{10}{1+t}\right)^2 \quad \left(\lim_{t\to\infty}\frac{10}{1+t} \approx 0\right)$ ≈ 500 tonnes per year	**Award 1** for correct solution

(v)	 	**Award 2** for correct solution **Award 1** for substantial progress towards solution
(c) (i)	Area under the curve represents the amount of carbon pollution. $\int_0^6 500-\left(\frac{10}{1+t}\right)^2 dt = \int_0^6 500-100(1+t)^{-2}dt$ $=\left[500t+100(1+t)^{-1}\right]_0^6$ $=\left[\left(500\times 6+100(1+6)^{-1}\right)-\left(100(1+0)^{-1}\right.\right.$ $=2914.285714...$ ≈ 2914 tonnes	**Award 1** for correct solution
(ii)	When $t=5$, $3200=2000e^{k\times 5}$ $e^{5k}=\frac{3200}{2000}$ $5k=\log_e\frac{8}{5}$ $k=\frac{1}{5}\log_e\frac{8}{5}$ $=0.09400072...$	**Award 1** for correct solution
(iii)	We need to find M when $t=10$ $M=2000e^{k\times 10}$ $=5120$	**Award 1** for correct solution

(iv)	We need to find t when $M = 4000$. $4000 = 2000e^{k\times t}$ $e^{kt} = 2$ $kt = \log_e 2$ $t = \frac{1}{k}\log_e 2$ $= 5\frac{\log_e 2}{\log_e \frac{8}{5}}$ $= 7.373849237.. \approx 7.4$ years $M = 2000e^{kt}$ $\frac{dM}{dt} = k2000e^{kt}$ $= kM$ $= \frac{1}{5}\log_e \frac{8}{5} \times 3200$ $= 300.8023227... \approx 301$	**Award 1** for correct solution